

Electrodynamics of coupled charge-density wave, 2D electron gas systems

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The combined inductive and coulombic coupling of an orbitally quantized two-dimensional electron gas to a one-dimensional charge-density wave (CDW) is shown to give rise to an anisotropic quantum fluid in which the Hall electric field, current and gradient of the CDW phase are all exponentially screened from within the bulk. The characteristic penetration depth is similar to the London penetration depth of relevance to superconductivity.

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Two-dimensional electron gas (2DEG) and charge-density wave (CDW) systems have each been explored extensively for their distinct electrodynamic properties. The formation of a Landau gap between the eigenstates of a 2DEG at high magnetic fields, for example, leads to a situation in which the diagonal conductivities σ_{xx} and σ_{yy} vanish while the off-diagonal Hall components $\sigma_{xy} = -\sigma_{yx}$ remain finite [1]. This leads to dissipationless conduction and also the quantum Hall effect upon considering interactions. The formation of a CDW, in contrast, leads to a gapped state that is electrically insulating, but can participate in charge transport upon being depinned from the crystalline lattice by an electric field \mathbf{E} [2].

In this paper, I show that the spatial coexistence of these two phenomena (i.e. a Landau quantized 2DEG and a CDW) occurring on different sections of Fermi surface, leads to a composite quantum fluid with electrodynamic properties that are entirely different from its constituents. Upon considering the combined inductive and coulombic coupling between the two, I obtain a pair of coupled equations that describe a situation in which the CDW undergoes local deformations of the phase ϕ in response to a divergent electric field $\nabla \cdot \mathbf{E}$ and current \mathbf{j} in the 2DEG. The anisotropic coupled fluid that results, involving the mutual exchange of quasiparticles between the 2DEG and states above and below the CDW gap, exponentially screens variations in \mathbf{E} , \mathbf{j} and ϕ from within bulk. The penetration depth λ , thus obtained, is similar to that of relevance to superconductivity [3]. I discuss the limitations of this effect, and possible experimental configurations in which it may have been observed.

To facilitate a comparison with existing materials [4, 5, 6, 7], I begin by considering a bulk anisotropic metal with separate one-dimensional (1D) electron-like and 2D hole-like quasiparticle dispersions. On choosing $\varepsilon_{1D} = \hbar v_F |k_x - k_F| + 2t_b \cos k_y b + 2t_c \cos k_z c$ and $\varepsilon_{2D} = \varepsilon_F - \hbar^2 (|k_x - \frac{\pi}{a}|^2 + |k_y - \frac{\pi}{b}|^2) / 2m + 2t_c \cos k_z c$ respectively (imposing the limits $t_b \ll \hbar\omega_c$ and $t_c \ll \hbar\omega_c$ to simplify subsequent derivations), I obtain the open and closed sections of Fermi surface depicted in Fig. 1(a). The 1D section is subject to CDW formation leaving the 2D section intact.

In the nearly-free electron limit, for which the residual

short range Coulomb repulsion is small compared to the electronic bandwidth, variations of \mathbf{j} and \mathbf{E} on the scale of the lattice parameters a , b and c are decoupled from long range variations of these quantities. In the present model, I consider variations in \mathbf{j} and \mathbf{E} over a characteristic distance λ (where $\lambda \gg 100$ nm) that greatly exceeds, not only the lattice parameters (of order 1 nm), but also the periodicity of the CDW ($2\pi/Q < 10$ nm) and the magnetic length ($l_m < 10$ nm at ~ 30 T). The electrostatic potential whose gradient defines the long range variations in $\mathbf{E}(\mathbf{r}) = -\nabla V(\mathbf{r})$ can therefore be considered a long range average that applies over distances $|\delta \mathbf{r}| \gtrsim 10$ nm, giving rise to a similar spatial average in the charge density defined by Poisson's equation $\nabla^2 V(\mathbf{r}) = -\rho(\mathbf{r})/\epsilon$ [8].

The electrodynamics can be further simplified by considering a metal-vacuum surface geometry with x normal to the surface at $x = 0$ like that in Fig. 1(b). The current $j_y(x)$ flows along y while the magnetic flux density $B_z(x)$ points along z . In this way, the spatial variation in all quantities is reduced to an effective 1D problem in which the x -dependence of the electrostatic potential is given by

$$V(x) = \frac{1}{2\epsilon} \int_0^\infty f(x-x') \rho(x') dx'. \quad (1)$$

For the bulk metal described above, $f(x) = |x|$ in contrast to the case of a single 2DEG for which $f(x) = -\ln|x|$ [9]. Each section of Fermi surface has the potential to yield separate contributions to the average charge density, enabling the total density to be expressed as the sum

$$\rho(x) = \rho_{1D}(x) + \rho_{2D}(x). \quad (2)$$

The presence of a magnetic field causes holes within the 2D section (assumed to be spinless) to undergo cyclotron motion, giving rise to a series of quantized Landau levels of energy $\varepsilon_{2D} = \varepsilon_F - \hbar\omega_c(n + \frac{1}{2})$ [10] where $n = 0, 1, 2, \dots$ and where $\omega_c = eB_z/m$ is the cyclotron frequency. Spatially varying electric fields modify this cyclotron motion. Because $E_x(x) = -\dot{V}(x)$ (the component of $\mathbf{E}(x)$ along x) varies on a length scale that exceeds the cyclotron radius, this first derivative of the electrostatic potential with respect to x causes the centers of the cyclotron orbits to drift at a steady velocity

$v_y(x) = \dot{V}(x)/B_z$ [1]; giving rise to the Hall current. The second derivative $\ddot{E}_x(x) = -\ddot{V}(x)$ modifies the harmonic oscillation of the holes in the Landau levels by contributing an additional electrostatic parabolic potential to their equation of motion [9]. The cyclotron frequency thus becomes

$$\omega^2(x) = \omega_c^2 - \frac{e\ddot{V}(x)}{m}, \quad (3)$$

where the ‘-’ sign is appropriate for holes [9]. Equation (3) can be verified semiclassically by equating $m d^2 \mathbf{r}' / dt^2$, where $\mathbf{r}' \equiv [x', y', z'] = [x_0 \cos \omega t, y_0 \sin \omega t + v_y(x)t, 0]$, to the total force $\mathbf{F} = e[B_z v'_y + \dot{V}(x) + \ddot{V}(x)x', -B_z v'_x, 0]$, where $\mathbf{v}' \equiv [v'_x, v'_y, v'_z] = d\mathbf{r}'/dt$. Higher derivatives of $V(x)$ perturb the motion without affecting either $v_y(x)$ or ω . The change from ω_c to ω modifies the eigenenergies of the Landau levels, so that $\varepsilon_{2D} = \varepsilon_F - \hbar\omega(n + \frac{1}{2})$. Because this change is very small, it is convenient to consider the first derivative $d\omega/d\ddot{V}(x) = 1/2\omega_c B_z$ of a Taylor expansion of $\omega(\ddot{V})$ with respect to $\omega = \omega_c$. This follows through to an inversely proportionate derivative $dD(x)/d\ddot{V}(x) = \bar{N}_{2D}/2\nu\omega_c B_z$ in the Landau level degeneracy $D(x) = \bar{N}_{2D}\omega/\nu\omega_c$. Here, \bar{N}_{2D} is the equilibrium number density of 2D holes, $\nu = F/B_z$ is the Landau level filling factor and F is the standard quantum oscillation frequency. A variation in $D(x)$ therefore corresponds to a variation in charge density

$$\rho_{2D}(x) = -2\alpha e\ddot{V}(x) + \frac{\beta \bar{N}_{2D}}{\nu \hbar \omega_c} \mu', \quad (4)$$

where $|\alpha| \gg 1$ is a dimensionless quantity that depends on the position of the chemical potential μ with respect to the highest occupied Landau level and μ' accounts for the possibility of a shift in the chemical potential (this has negligible affect at integer filling but may be important at half-integer filling). At integer Landau level filling factors (when μ is situated between two Landau levels), $\alpha_{\text{int}} = e\bar{N}_{2D}/4e\omega_c B_z$ [9]. At half-integer filling factors (when μ is situated in the middle of a Landau level), however, $\alpha_{\text{half}} = -\infty$. The latter unphysical result reflects the fact that the delta function lineshapes of the Landau levels in an ideal 2D metal cause $\rho_{2D}(x)$ to jump by $e\bar{N}_{2D}/2\nu$ for an infinitesimal change in ω . The presence of a finite density of states at μ introduces a feedback effect that becomes excessive at half-integer filling factors. A more physical result is obtained upon considering a finite relaxation time τ , which causes the Landau levels to become broadened into Lorentzians [10]. In this case, $\alpha = e\bar{N}_{2D}(1 - \beta)/4e\omega_c B_z$, with $\beta_{\text{int}} \approx 4/\pi\omega_c\tau$ and $\beta_{\text{half}} \approx 2\omega_c\tau/\pi$ at integer and half-integer filling factors respectively.

It is clear that in the case of a layered metal without a CDW (in which case $\rho_{1D} = 0$), the insertion of Equations (4) and (2) into Equation (1) yields a result only for the uninteresting situation where $\dot{V}(x) = 0$. This is one of the reasons why charge tends only to accumulate beyond the surface in bulk metals, in contrast to

single layer 2DEGs [9]. In order to understand how the presence of a CDW on the 1D Fermi surface section can change this situation, it is helpful to consider a Ginzburg-Landau model for CDW excitations [2]. If I ignore high energy amplitude excitations, derivatives involving time and spin, the local free energy of a CDW is given by

$$\Phi = \Phi(0) + g_{1D} \left(-\frac{\Delta^2}{2} + \frac{\hbar^2 v_F^2}{4} \dot{\phi}^2(x) \right), \quad (5)$$

where g_{1D} is the density of 1D electronic states, $\Phi(0)$ is the free energy prior to formation of the CDW and $\phi(x)$ is the local CDW phase. Here, I define Q as the component of the CDW nesting vector \mathbf{Q} parallel to v_F , while $Q'(x) = \partial\phi(x)/\partial x \equiv \dot{\phi}(x)$ is the extent to which it departs from equilibrium. A change in Q from its equilibrium value causes the midpoint of the charge gap 2Δ to shift by an amount $\mu'(x) = \hbar v_F k_F Q'(x)/Q = Q'(x)\bar{N}_{1D}/Qg_{1D}$ with respect to μ , where \bar{N}_{1D} is the mean number density of 1D carriers [12]. This gives rise to a charge

$$\rho_{1D}(x) = -eg_{1D}\mu'(x); \quad (6)$$

not to be confused with the underlying CDW charge modulation that oscillates with a much shorter periodicity $2\pi/Q \ll l_c$, where $l_c = \sqrt{2\hbar F/eB_z^2}$ is the 2DEG cyclotron radius. Upon combining Equations (1), (2), (4) and (6), I obtain $\ddot{V}(x) = -eg_{1D}\mu'(x)/2e(\alpha + 1)$. On further making the approximations $1/\alpha \approx 0$ and $Q \approx 2k_F$, this becomes

$$\ddot{V}(x) = -\frac{(\eta + \beta)v_F B_z}{\nu(1 - \beta)} \dot{\phi}(x), \quad (7)$$

where $\eta = g_{1D}/g_{2D}$ is the ratio of the mean density of electronic states for the two Fermi surface sections. By itself, equation (7) implies that the charge density of the 2D component of the electronic structure can vary arbitrarily throughout the bulk because it can always be compensated by a charge density of opposite sign from the CDW. A small fraction of excess charge $1/\alpha$ nevertheless remains in order to enable E_x to vary within the bulk while satisfying Poisson's equation. This exchange of charge degrees of freedom is the primary origin of the coupling between the two subsystems.

Next, I consider the inductive coupling between the 1D and 2D Fermi surface sections resulting from current flow. The Hall current $j_y(x) = (eN_{2D}/B_z)\dot{V}(x)$ affects the CDW by changing the local magnetic field $H_z(x)$, where, according to Maxwell's equations, $j_y(x) = -\partial H_z(x)/\partial x \equiv -\dot{H}_z(x)$. It is well established that a change in $H_z(x)$ leads to a change in the chemical potential $\mu'(x)$ of the eigenstates [11]. These are related via the derivative

$$\frac{\partial \mu'(x)}{\partial H_z(x)} = \frac{\mu_0 \hbar e F}{\gamma m B_z} \quad (8)$$

where $B_z = \mu_0(H_z + M_z)$ and where $\gamma_{\text{int}} = \eta$ and $\gamma_{\text{half}} = -1$ at integer and half integer filling factors respectively;

the sign change with respect to Reference [12] accounts for the fact that I consider holes rather than electrons in the present paper. Given the relation $\mu'(x) = \hbar v_F \dot{\phi}(x)/2$ identified above, upon taking its spatial derivative and combining cross derivatives $\partial\dot{\phi}(x)/\partial x = \partial\dot{\phi}(x)/\partial\mu'(x) \times \partial\mu'(x)/\partial H_z(x) \times \partial H_z(x)/\partial x$, I obtain

$$\dot{V}(x) = -\frac{\gamma m v_F B_z}{2\mu_0 \nu e^2 \bar{N}_{2D}} \ddot{\phi}(x). \quad (9)$$

Equations (7) and (9) constitute a pair of coupled equations that describe the variation of the electric field, CDW phase gradient and current density with x . On taking the spatial derivative of Equation (7) in order to combine them, I obtain

$$\dot{V}(x) = \left(\frac{m}{2\mu_0 e^2 \bar{N}_{2D}} \right) \ddot{V}(x) \quad (10)$$

where the scalar quantities γ , β and η conveniently cancel on imposing the limit $\omega_c \tau \gg 1$. Equation (10) implies that the Hall electric field, the Hall current and the phase derivative of the CDW are all exponentially screened from within bulk so that $E_x(x) = E_{x,0} e^{-x/\lambda}$, $j_y(x) = j_{y,0} e^{-x/\lambda}$ and $\dot{\phi}(x) = \dot{\phi}_0 e^{-x/\lambda}$. The characteristic penetration depth

$$\lambda = \sqrt{\frac{m}{2\mu_0 e^2 \bar{N}_{2D}}}; \quad (11)$$

is strikingly similar to the London penetration depth of relevance to superconductivity [3].

The electrodynamics can be rather succinctly described by a quantum fluid in which the canonical momentum

$$\mathbf{p} = e\mathbf{A} + \frac{m}{2} \frac{\partial \mathbf{r}}{\partial t} + e(\mathbf{r} \times \mathbf{B}_0) \quad (12)$$

consists of a superfluid-like component coupled to a term dimensionally equivalent to a Chern-Simons gauge field [1]. This comes as no coincidence. The first two terms on the right-hand-side of Equation (12) originate from the existence of a relationship between the current density and the phase of a wavefunction; a property it shares with superconductivity [3]. In the present system it is the CDW phase [2] rather than the phase of a superconducting order parameter [3] that is of relevance. The third term on the right-hand-side of Equation (12) reflects the fact that the current in the present system is always encumbered by a transverse Hall electric field, unlike a supercurrent [3]. This results from the fact that the Landau levels states in which the holes reside accommodate the presence of a large steady background magnetic flux density \mathbf{B}_0 deep within the bulk; i.e $\lambda \gg x$. On setting $\mathbf{p} = 0$, for the geometry considered in Fig. 1, this becomes

$$\mathbf{A} = [A_x, A_y, A_z] = [-yB_0, -\mu_0 \lambda^2 j_y, 0]. \quad (13)$$

It is clear from Equation (13) that the current must be non-accelerative (i.e. $\partial j_y / \partial t = 0$) in order to reproduce

Equation (10), further implying that $\sigma_{xx} = \sigma_{yy} = 0$. Thus, upon taking the time derivative of Equation (13), I obtain the Hall effect;

$$E_x = -2\omega_c \lambda^2 j_y. \quad (14)$$

The curl of Equation (13), in contrast, yields the London [3] equation governing the variation of currents

$$H_z = -\lambda^2 \dot{j}_y + \frac{B_0}{\mu_0}, \quad (15)$$

albeit in the presence of a background magnetic flux density B_0 . Equation (10) can then be obtained upon combining the curl of Equation (15) with Maxwell's equation, $\dot{H}_z = -j_y$, and substituting j_y from Equation (14).

There are two ways that $\sigma_{xx} = \sigma_{yy} = 0$ could be achieved with normal electrons in order for the above electrodynamics to come into effect. First, conditions amenable to the realisation of the quantum Hall effect could be realised at integer filling factors [1]. Second, the conductivity can vanish if the lengthscale over which \mathbf{E} varies is shorter than the mean free path l . For example, an electron or hole incident upon the edge from within the bulk might be accelerated by the electric field within a distance $\approx \lambda$ only to be decelerated by the same electric field upon reflection. However, a more fundamental reason for the vanishing conductivity may emerge from the physics of the anisotropic coupled state itself. Equations (7) and (9) imply that as a cyclotron orbit drifts through the system (in the vicinity of the edge), the inductive and coulombic coupling causes the CDW to become distorted in its wake. The charge of the hole in the cyclotron orbit is therefore effectively screened by that of the deformed CDW, causing the current-induced anisotropic coupled state to possess zero net charge. For this reason, the coupled state does not respond to longitudinal electric fields giving rise to $\sigma_{xx} = \sigma_{yy} = 0$, but drifts only orthogonal to a transverse electric field. In this way, the resulting electrodynamics is phenomenologically similar to the quantum Hall effect, except that here the Hall electric field and the Hall current are confined within a distance λ of the surface, and realisation of $\sigma_{xx} = \sigma_{yy} = 0$ is not directly dependent on filling factor ν . Furthermore, when a transport current flows along the surface of the material, ones ability to measure the Hall potential difference will be extremely sensitive to the sample geometry, the current path and the arrangement of the contacts. Full realisation of a dissipationless conducting state requires the penetration depth to lie within the range $l_c < \lambda < d$, where d is mean distance of CDW pinning sites from the sample surface. If $\lambda < l_c$, for example, the cyclotron motion will become anharmonic and the physics will instead be dominated by phenomena associated with edge states [1]. If, however, $\lambda > d$, the CDW will be mostly unable to deform, enabling electric fields and currents to penetrate into the bulk as in conventional metals. Realisation of the inequality $\lambda < l_c$ ultimately requires electronic states of the closed Fermi surface section to vary

strongly with ε , which is most easily realised in the case of a 2D electronic dispersion in strong magnetic fields. Realisation of $\lambda > d$, however, requires the metal to have a high degree purity.

Charge-transfer salts of the type α -(BEDT-TTF)₂MHg(SCN)₄ may be the most likely candidates to fulfill these criteria. They possess 2D and 1D electronic dispersions like that considered in the model [13], and the 1D sections are also thought to undergo a CDW instability at high magnetic fields where the effects of Landau quantization of the 2D sections are clearly observed [14]. These materials are also shown to exhibit a variety of unusual galvanomagnetic effects. A pronounced increase in conductivity by as much as two orders of magnitude is observed at low temperatures in some crystals, having a behaviour closely resembling that of an inhomogeneous superconductor [4]. This effect is observed to become weaker at half-integral filling factors. The Hall resistance is also measured to exceed the longitudinal resistance [5], with a conduction path that is profoundly sensitive to contact geometry [6]. Furthermore, contactless high frequency measurements yield an inductive response, part of which can be explained by dissipationless surface

currents [7]. A second contribution to the total current has been shown to arise from the interaction between the composite CDW-2DEG system and CDW pinning sites in the absence of an electric field [12], although these currents are purely magnetic \mathbf{j}_m in contrast to the electrical ones discussed in the present model.

In summary, I have shown that inductive and coulombic coupling between a CDW and 2DEG establishes the existence of a quantum fluid with significantly different electrodynamics compared to the entities from which it is composed. Under ideal conditions, the Hall electric field, current and gradient of the CDW phase are all exponentially screened from within the bulk, with a characteristic penetration depth similar to the London penetration depth. The present model considers variation of the electromagnetic fields only with respect to x in order to simplify the derivation, but the same electrodynamics is expected to extend to more general geometries.

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FIG. 1: (a) A sketch of the Fermi surface according to the model, consisting of open and closed sections. (b) A portion of the surface of a layered material with this Fermi surface of spatial dimensions greatly exceeding the penetration depth λ (see text).

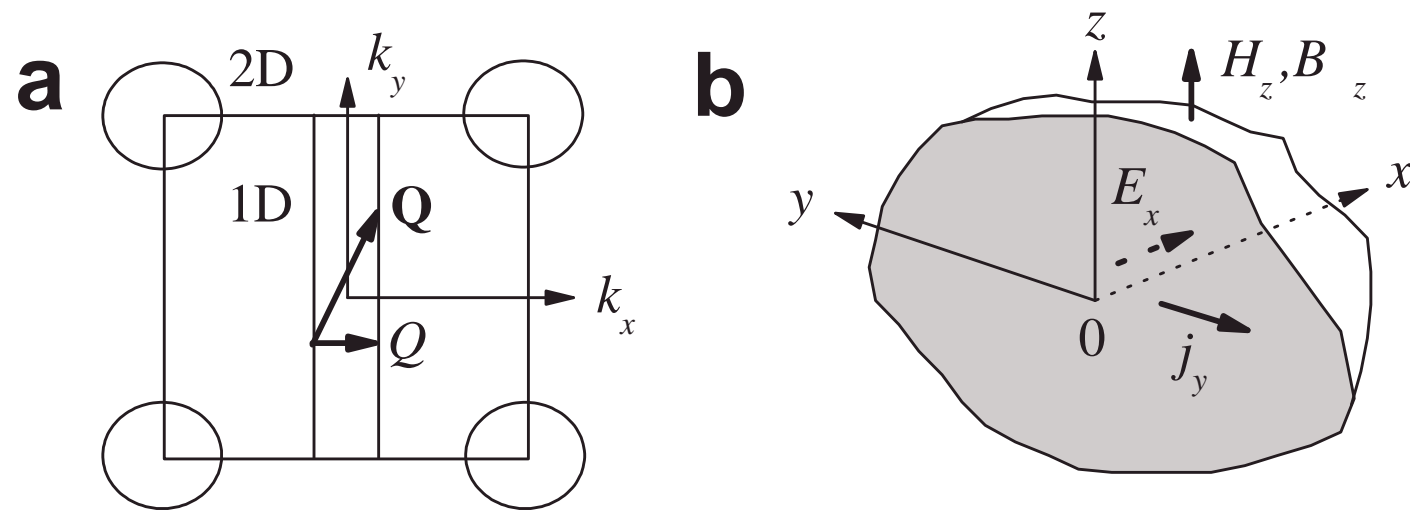


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